Nuclear Energy

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Block Ia, 2020/2021

1 The Building Blocks of Matter

1.2 The Nucleus

The nucleus is the massive centre of the atom in which almost all of the atomic mass is concentrated. It consist of neutrons and protons, these in turn are made up out of quarks, which are held together by gluons.

Particle	Symbol	Charge	Mass		
		[C]	[kg]	[u]	[MeV/c^2]
Proton	p	$+1.6021762 \times 10^{-19}$	1.67262×10^{-27}	1.007276	938.272
Neutron	n	0	1.67493×10^{-27}	1.008665	939.565
Electron	e	$-1.6021762 \times 10^{-19}$	$9.10938356 \times 10^{-31}$	5.485799×10^{-4}	0.511099

An atom can be identified by a couple of numbers; its amount of neutrons is given by N, the amount of protons by Z and, if neutral, the amount of electrons will also be Z. Z is also know as the **atomic number**. Together N and Z give the **mass number**, or the number of nucleons:

$$A = Z + N$$

The correct nucleus notation is given by:

$$^{A}_{Z}X$$

Removing or adding one *proton* in a nucleus *transforms* an element into another. If one adds or removes one neutron, the number of peripheral electrons remains the same and therefore the chemical properties of the atom are not changed. This means that such an atom should remain in the same place of the Periodic Table of the Elements.

- **Isotopes** have same number of protons (Z) but a different number of neutrons (N).
- **Isobars** have the same mass number (A), but different amounts of protons and neutrons.
- **Isotones** have the same number of neutrons (N) and a different number of protons (Z).

1.3 The Periodic Table of the Elements

There are 98 naturally occurring kinds of atoms (elements). Scientists in labs have been able to artificially produce about 20 more. These are organized in the Periodic Table of the Elements.

1.4 Nuclear Size and Density

In first approximation, protons and neutrons in the nucleus are packed to form a roughly spherical region with radius R given approximately by:

$$R \approx r_0 A^{1/3}$$

with $r_0 = 1.2 \times 10^{-15}$ m= 1.2 fm. The volume of the nucleus then is given by

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_0^3 A.$$

So, if nucleus A has twice the radius of nucleus B, then nucleus A will have eight times (2^3) as many nucleons as nucleus B.

From this follows that the diameter of an atom is in the range of 10^{-10} m, the diameter of an nucleus is in the range of 10^{-14} m and the diameter of an proton is in the range of 10^{-15} m.

1.5 The Nuclear Force and the Diagram of Nuclei

The diagram of nuclei is a diagram of the number of protons Z as a function of the number of neutrons N for the currently known nuclei. It also indicates the stable nuclei with black squares and indicates what type of nuclear decay unstable nuclei will go through with different colors. The curve going through the stable nuclei is called *the stability curve*.
The figure also shows that the light nuclei ($A < 20$) generally contain almost an equal number of protons and neutrons, while for heavier nuclides the number of neutrons N is always greater than that of the protons Z, and the neutron excess tends to grow when Z increases. This is because the neutrons act as a kind of glue that keeps repelling protons together. The greater the repelling charge, the more glue is necessary.

Protons and neutrons are bound together in the nucleus by a fundamental force called nuclear force. The nuclear force is different from both the electric force and the gravitational force: at distances typical of neutrons and protons in nuclei ($\sim 10^{-15}$ m), the nuclear force is much stronger than both (for this reason it is also called the strong force), but decreases more rapidly with increasing the distance between nucleons. This short range of this force is the cause that nuclei with more than 83 protons become unstable.

1.6 Nuclear Masses and Mass Defect

A nucleus with Z protons and A nucleons has a mass $M(Z, A)$ slightly smaller than the sum of the masses of the A nucleons that it contains:

$$M(Z, A) < ZM_p + NM_n.$$

The missing mass,

$$\Delta m = ZM_p + NM_n - M(Z, A),$$

is called the *mass defect* and indicates the degree of binding of protons and neutrons in the nucleus. This fact is quite general: the mass of any bound system M is less than the sum of the masses of its constituents when they are separated, because one has to provide energy to separate them.

It is convenient to include the mass of the Z atomic electrons and use the atomic masses since they can be measured to a considerably higher precision than nuclear masses.

$$\Delta m = ZM_H + NM_n - M_A(Z, A)$$

Here $M_H = (M_p + m_e)$ is the mass of the hydrogen atom and $M_A(Z, A) = M(Z, A) + Zm_e$ is the mass of the atom with Z electrons whose nucleus contains A nucleons.

1.7 The Mass-Energy Equivalence

Energies are usually measured in *electronvolt* (eV), where 1 eV = 1.602×10^{-19} J. Masses are usually expressed in terms of the atomic mass unit (u), this unit is equivalent to 1/12th of the mass of neutral ^{1}2C. Here 1 u = 1.66×10^{-27} kg.

Masses can also be expressed in terms of energy units, because according to relativity, the energy of a body at rest, E_0, is related to its mass M by the relationship

$$E_0 = Mc^2.$$

This relation is called *the equivalence of mass and energy* or *mass-energy equivalence*. The conversion factor between energy (MeV) and mass (u) follows from this relation: 1 u = 931.487 MeV/c^2.

1.8 Nuclear Binding Energy

The **nuclear binding energy** ($B(Z, A)$) is the energy required to split a nucleus into separated nucleons or, conversely, the energy that would be liberated when Z protons and N neutrons combine to form a nucleus. It is defined as the difference between the masses of the nuclear constituents and the nuclear mass:

$$\begin{aligned} B(Z, A) &= \Delta mc^2 \\ &= \left[ZM_p + NM_n - M(Z, A)\right] c^2 \\ &= \left[ZM_H + NM_n - M_A(Z, A)\right] c^2 \end{aligned}$$

The neutron separation energy of nucleus ${}^{A}_{Z}X_N$ is given by

$$S_n = B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z}X_{N-1}).$$

The proton separation energy of nucleus ${}^{A}_{Z}X_N$ is given by

$$S_p = B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z-1}X_N).$$

1.9 The Nuclear Valley of Stability

The unstable nuclei have a proportion of protons and neutrons that does not allow them to be energetically stable. As a consequence they undergo a transition to a stable configuration by transmuting to another nucleus, changing protons to neutrons, or neutrons to protons, while keeping the same atomic mass number. This way they move to the valley of stability.

Drip lines delineate the boundary between bound and unbound nuclei. The proton dripline ($S_p = 0$) is quite well known, up to $Z \sim 82$, but the neutron dripline ($S_n = 0$) is not really known, only up until $Z = 10$ (neon).

1.10 Nuclear Reactions

The **Q-value** is the amount of energy absorbed or released by a reaction or decay. It is defined as the difference between the final and initial kinetic energies

$$\begin{aligned} Q &= T_b + T_Y - T_a \\ &= \left[(M_a + M_X) - (M_b + M_Y)\right] c^2 \end{aligned}$$

When Q is positive, the reaction is **exothermic**, or *exoergic*, since it releases kinetic energy by the conversion of a portion of the rest mass into kinetic energy. Conversely, when Q is negative, the reaction is **endothermic**, or *endoergic*.

2 Radioactivity and Penetrating Power of Nuclear Radiation

2.1 Nuclear Decay

2.1.1 α-Decay

For the heaviest nuclei whose average nuclear binding energy is relatively low, some of their nucleons can go towards a higher binding energy by emitting a helium nucleus, this is α-decay. The formula of this reaction is given by:

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He} \qquad \text{or} \qquad {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha $$

The energy, Q, that is released is distributed between the α-particle and the daughter nucleus:

$$ Q_\alpha = \left[M(Z,A) - M(Z-2,A-4) - M({}^{4}\text{He})\right]c^2 $$

2.1.2 β^--Decay

A β^--decay (beta-minus decay) occurs when the ratio of neutrons to protons in the nucleus is too high. In this case, an excess neutron transforms into a proton and an electron plus a neutral particle of negligible mass, called antineutrino.

$$ n \rightarrow p + e^- + \bar{\nu} $$

The proton stays in the nucleus and the electron is ejected. The formula of this reaction is given by:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu} $$

The energy, Q, that is released is given by the mass difference between the parent and the daughter atom multiplied by c^2:

$$ Q_{\beta^-} = \left[M(Z,A) - M(Z+1,A)\right]c^2 $$

2.1.3 β^+-Decay

A β^+-decay (beta-plus decay) occurs when a proton inside a nucleus converts into a neutron by emitting a positron (i.e., a positive electron) and a neutral particle called neutrino.

$$ p \rightarrow n + e^+ + \nu $$

The parent nucleus transforms into a daughter nucleus having the same mass number A and the atomic number Z smaller by one unit:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu $$

The total energy available for the β^+-decay is

$$ Q_{\beta^+} = \left[M(Z,A) - M(Z-1,A) - 2m_e\right]c^2 $$

2.1.4 Electron Capture

For nuclei in which the ratio of protons to neutrons is too high electron capture will occur. This is a decay process in which an atomic electron interacts with the nucleus, where it combines with a proton, forming a neutron and a neutrino. The neutrino is ejected from the atomic nucleus:

$$ p + e^- \rightarrow n + \nu $$

The atom changes a proton into a neutron, so that it changes from one element to another. However, the total number of nucleons remains the same.

$$ {}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + \nu $$

The energy available in the capture process is given by

$$ Q_{e^-} = \left[M(Z,A) - M(Z-1,A)\right]c^2 $$

2.1.5 γ-Decay

A γ-decay (gamma decay) occurs when a nucleus that is in an excited state (that is, in a configuration with energy higher than the ground state) makes a transition to either an excited state with lower energy than the initial one or to the ground state (we recall that this is the most stable configuration of the nucleus). It does this by emitting a photon.

$$ {}^A_Z X^* \rightarrow {}^A_Z X + \gamma $$

2.1.6 Internal Conversion

Internal conversion is a radioactive decay process in which an excited nucleus, instead of emitting a gamma ray, de-excites by knocking out one of the electrons in the atom. The electron is emitted with a well-defined energy, the same energy that a gamma ray would have had in the same decay process. Thus, in an internal conversion process, a high-energy electron is emitted from the radioactive atom, not from the nucleus. For this reason, the high-speed electrons resulting from internal conversion are not beta particles, since the latter come from beta decay, where they are produced in the nuclear decay process.

2.1.7 Nucleon Emission

Highly excited neutron-rich or proton-rich nuclei, formed as the product of other types of decay, can occasionally lose energy by way of neutron and proton emission, resulting in a transition from one isotope to another of the same element, or from a nuclide of one element to a nuclide of another element, respectively.

2.1.8 Spontaneous Fission

Spontaneous fission is a form of decay where the nucleus disintegrates into other nuclei that are not well defined, but rather correspond to a range of fragments, often 2, sometimes 3, of the original nucleus. This generally leads to the emission of gamma rays, neutrons, and beta particles and it the result of direct competition between attractive nuclear force and Coulomb repulsion.

2.2 The Radioactive Decay Law

The radioactive decay law describes the statistical behaviour of large number of nuclides. The delay of a nucleus is a random process, and the probability of decay (λ) is constant.

$$ N(t) = N_0 e^{-\lambda t} $$

The **half-life** or **half-period** ($\tau_{1/2}$) is the time interval over which, on average, half of the radionuclides present in the radioactive material disintegrate. This half-life is given by:

$$ \tau_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. $$

The **mean-life** of an isotope is its average survival time, which is given by:

$$ \tau = \frac{1}{\lambda} = \frac{\tau_{1/2}}{\ln 2} = \frac{\tau_{1/2}}{0.693}. $$

This also gives that

$$ \tau_{1/2} = 0.693 \times \tau. $$

2.2.1 Activity

The **activity** is defined as the number of disintegrations per unit time. It is given by

$$A = \lambda N$$
$$= A_0 e^{-\lambda t}.$$

Its unit is the Becquerel [Bq] or Curie [Ci], where 1 Ci= $3.7 \times 10^1 0$ Bq.

2.3 Radioactive Families

A decay chain is a sequence of radioactive decay processes, in which the decay of one nucleus creates a new nucleus that is itself radioactive, and this sequence continues until eventually a stable nucleus is reached. The group of nuclides within a series of decays is called *radioactive family*.

2.4 Penetrating Power of Nuclear Radiation

Radiation from radioactive nuclei produces specific physical effects when it crosses materials. In particular, the interaction between radiation and matter can result in damages to the crossed materials.

Alpha particles can travel only a few centimetres in the air and can be stopped by a sheet of paper or a layer of skin. Beta particles can travel metres in the air and can be stopped by a sheet of metal of a few millimetre thickness. Gamma rays, X-rays, and neutrons are more penetrating; thick barriers of dense metal like lead or thick walls of concrete best stop gammas, while neutrons are best shielded by thick layers of concrete or by materials rich in hydrogen atoms, such as water or paraffin. Neutron absorbers like boron or cadmium are also used for their ability to capture neutrons.

The penetration range does not only depend on the material and the type of radiation, but also on the energy of the particles.

3 Nuclear Reactions and Fission

3.1 Nuclear Collisions

A nuclear reaction can be defined as a collision between two nuclei that produces a change in the nuclear composition and/or in the energy state of the interacting nuclei.
When a collision occurs between two nuclei or a particle and a nucleus, several different reactions may take place:

1. One speaks of **scattering** of an incident nucleus/particle colliding with a target nucleus when the incident projectile is found among the products of the reaction.
    - The scattering is **elastic** if the interacting nuclei remain intrinsically unchanged in the collision (i.e. the initial and final products are identical) and no other particles are produced.
    - The scattering is **inelastic** if the target nucleus (or the projectile) becomes excited, or breaks up.
2. One speaks of nuclear **transmutation** if there is a rearrangement of nuclear constituents between the colliding nuclei.
3. One speaks of **absorption** if the incident neutron enters target nucleus
    - We speak of **radiative capture** when the incident neutron enters target nucleus forming a compound nucleus. Compound nucleus then decays to its ground state by gamma emission
    - We speak of **fission** when the incident neutron enters target nucleus forming a compound nucleus which then splits into a few lighter fragments releasing a large amount of energy and a certain number of free neutrons.

3.2 Cross Section

An important quantity for the description and interpretation of a nuclear reaction is the so-called cross-section, which quantifies the probability of the occurrence of the reaction.

The **macroscopic cross section** is the probability of a given reaction occurring per unit travel of the neutron

$$\sum = N\sigma.$$

Here N is the **number density** of nuclei in a material in cm^{-3} and σ is the microscopic cross section. The unit of the cross section is the barn, where $1\ \mathrm{b} = 10^{-24}\ \mathrm{cm}^{-2}$

The reaction rate of a reaction is given by

$$R = \phi N\sigma = \phi \sum$$

The atom (or number) density is given by

$$N = \frac{\rho N_A}{M}$$

And the mean free path is given by

$$\lambda = \frac{1}{\sum}$$

3.3 The Fission Process

Nuclear fission is a reaction in which a nucleus of a heavy element splits into a few lighter fragments (usually two, so-called binary fission), releasing a large amount of energy and a certain number of free neutrons (typically two or three).

(Z, A) splits into (Z_1, A_1) and (Z_2, A_2) with $Z = Z_1 + Z_2$ and $A = A_1 + A_2$ in first approximation. The energy released in this process is the difference between the final and initial binding energies

$$Q = B(A_1, Z_1) + B(A_2, Z_2) - B(A, Z).$$

Fission mechanism A nucleus goes through some shapes before/ during fission:

- Spherical nucleus: Coulomb energy is greatest, surface energy least
- Cigar nucleus: nucleus excited by some stimulus. Strong deformation of compound nucleus: more surface energy, less Coulomb energy
- Peanut nucleus: nucleus experiences vibrations (elongation and compression), gets saddle point configuration. Coulomb energy decreases, surface energy increases.
- Deformation beyond saddle point: neck disappears and nucleus divides into two excited fragments at scission point. Large Coulomb repulsion accelerates fragments away from each other

3.3.1 Spontaneous Fission

For spontaneous fission to occur, the nucleus must undergo a deformation of its shape. In its deformed state, two forces are acting on the nucleus, increased surface energy and Coulomb repulsion between fragments. This surface tension creates some form of potential barrier, so the fission fragments must undergo quantum tunnelling through this barrier. The probability of tunneling is proportional to

$$\exp\left\{-2\ell\sqrt{\frac{2m(U_0 - E)}{\hbar^2}}\right\}$$

3.3.2 Induced Fission

This process can be started by striking the nucleus with a neutron, then we have neutron induced fission. Neutrons can react down to very low energies. Neutron capture leaves the nucleus in an exited state and this excited nucleus can decay by γ-emission or by fission.

Resonances Resonances occur in direct and compound reactions. Each resonance corresponds to an excited nuclear state in the continuum. If the sum of energy of the projectile and energy of target nucleus is equal to excitation energy (state) in compound nucleus, resonance can be created and peak occurs in the cross section.

Fissionable nuclei A **fissile** material will undergo fission when bombarded by neutrons of any energy. A **fertile** material will capture a neutron, and transmute by radioactive decay into a fissile material. Fertile isotopes may also undergo fission directly, when bombarded by a high energy neutron (MeV range).

3.4 Fission Products

The fission of any particular nucleus can produce many different combinations of fission fragments. The number of nucleons (protons + neutrons) is conserved, but there are many combinations.

3.5.1 Energy Released by Nuclear Fission

The average energy released by fission is about 200 MeV.

3.6 The Chain Reaction

Fission can be a self-sustained process, that once started, needs no additional agents to keep it going. This is possible if the number of neutrons (ν) released per fission is (sufficiently) greater than one. This needs to be the case to compensate for neutron losses (capture reactions and leakage)

3.7 The Slowing Down of Neutrons

Neutrons can be slowed down to thermal energies via collisions. The relation between energy of elastically scattered neutron and neutron incident energy

$$E_s = \alpha E_i, \text{ with } \alpha = \left(\frac{A-1}{A+1}\right)^2$$

α is the collision parameter, i.e measure of an element's efficiency to slow down neutrons The number of collisions needed to travel from high energy to low energy is given by

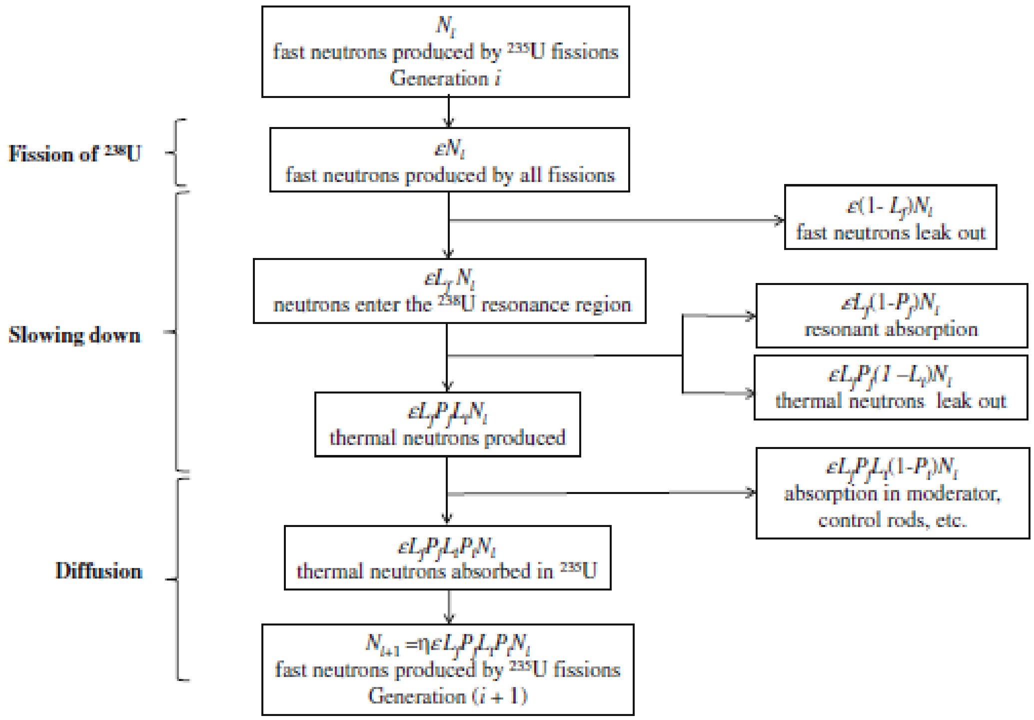
$$N = \frac{\ln E_{high} - \ln E_{low}}{\xi}$$

Here ξ is a measure of energy loss per collision $\xi = 1 + \frac{\alpha}{1-\alpha}\ln\alpha$ The fraction of energy retained by a neutron in a single elastic collision is a constant for a given material, so ξ is also a constant.

3.8 The Thermal Nuclear Reactor

3.9 The Physics of a Thermal Nuclear Reactor

The sequence of processes from the production of a neutron to its final absorption on a fissile nucleus is usually called a generation of neutrons, and the time required for this sequence to take place is called generation time. A schematic representation of the life-cycle of neutrons in a thermal reactor is given here



We start with an arbitrary number N_i of fast neutrons. Multiplying this with the *fast-fission factor*, ε, gives us the fast neutrons produced by all fissions εN_i. These will diffuse in the moderator, and slow down regularly to the thermal energy, during this cooling down some neutrons are lost in radiative capture of by escaping the reactor (as fast *or* as thermal neutrons).

P_f (p) is the fraction of fast neutrons that are *not* captured

L_f is the fraction of fast neutrons that do *not* escape from the reactor

P_t (f) is the fraction of thermal neutrons that are *not* captured

L_t is the fraction of thermal neutrons that do *not* escape the reactor

So from this follows that

$\varepsilon(1 - L_f)N_i$ is the fraction of fast neutrons that do escape from the reactor

$\varepsilon L_f(1 - P_f)N_i$ is the fraction of fast neutrons that are *not* captured

$\varepsilon L_f P_f(1 - L_t)N_i$ is the fraction of thermal neutrons that do *not* escape the reactor

$\varepsilon L_f P_f L_t(1 - P_t)N_i$ is the fraction of thermal neutrons that are *not* captured

If η is the average number of neutrons emitted per thermal neutron absorbed (in fission and other reactions) in the fuel, at the end of the cycle there will be $N_{i+1} = \eta\varepsilon L_f P_f L_t P_t N_i$ fast neutrons available for a new cycle. The number η is called fission factor. It is smaller than the average number of neutrons produced per fission, ν, because not all neutrons that are absorbed by the uranium produce fissions. The two quantities are related by the following relationship

$$\eta = \nu\frac{\sigma_f}{\sigma_f + \sigma_r},$$

where σ_f is the fission cross section and σ_r is the absorption cross section for thermal neutrons by all processes, excluding fission.

From the above it follows that in a cycle the number of neutrons increases by a factor of

$$k = \eta\varepsilon L_f P_f L_t P_t \text{ (six-factor formula)}$$

This parameter is called the **effective multiplication factor**. For a finite reactor we have

$$k < 1 \text{ subcritical}$$
$$k = 1 \text{ criticality condition}$$
$$k > 1 \text{ supercritical}$$

The **infinite multiplication factor**, k_∞ is only made up of the factors that don't depend on the geometry and size of the reactor, but on the composition and nature of the fuel, the moderator and the other materials.

$$k_\infty = \eta \varepsilon P_f P_t \text{ (four-factor formula)}$$

3.10 Reactor Control and Delayed Neutron Emission

The number of neutrons present at time t is given by

$$N(t) = N_0 k^{t/\tau} = N_0 e^{(k-1)t/\tau}$$